Partial Derivatives with TI-Nspire™ CAS

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TI-Nspire CAS does not have a function to calculate partial derivatives. Nevertheless, recall that to calculate a partial derivative of a function with respect to a specified variable, just find the ordinary derivative of the function while treating the other variables as constants. For example, suppose we have the function \( g(x, y) = 2x + 2y \). To find the partial derivative of \( g \) with respect to \( x \), treat \( y \) as a constant and take the derivative of \( g(x, y) \) with respect to \( x \): \( \frac{d}{dx}(g(x, y)) \). Likewise, to find the partial derivative of \( g \) with respect to \( y \), treat \( x \) as a constant and take the derivative of \( g(x, y) \) with respect to \( y \): \( \frac{d}{dy}(g(x, y)) \).

Thus, to calculate the partial derivative of a function of two or more variables, use the `derivative()` function or the derivative template: `derivative(f(x,y),x)` or \( \frac{d}{dx}(f(x,y)) \) calculates the first partial derivative of \( f(x, y) \) with respect to \( x \) and `derivative(f(x,y),y)` or \( \frac{d}{dy}(f(x,y)) \) calculates the first partial derivative of \( f(x, y) \) with respect to \( y \).

**Example**

a. Define functions for and calculate the first partial derivatives of \( f(x, y) = \sqrt{x^2 + y^2} \). Define the functions to facilitate calculating the second partial derivatives or to evaluate the partial derivatives at a given point \((x, y)\).

Define \( f(x, y) \):

\[
f(x, y) := \sqrt{x^2 - y^2}
\]

Done

Define a function for \( \frac{\partial f}{\partial x} = f_x \) and display the definition:

\[
f_x(x, y) := \frac{d}{dx}(f(x, y))
\]

Done

Define a function for \( \frac{\partial f}{\partial y} = f_y \) and display the definition:

\[
f_y(x, y) := \frac{d}{dy}(f(x, y))
\]

Done
b. Define functions for and calculate the four second partial derivatives of $f(x, y)$:

Define a function for $\frac{\partial^2 f}{\partial x^2} = f_{xx}$ and display the definition:

$$f_{xx}(x, y) := \frac{d}{dx}(f_x(x, y))$$

Define a function for $\frac{\partial^2 f}{\partial x \partial y} = f_{xy}$ and display the definition:

$$f_{xy}(x, y) := \frac{d}{dy}(f_x(x, y))$$

Define a function for $\frac{\partial^2 f}{\partial y^2} = f_{yy}$ and display the definition:

$$f_{yy}(x, y) := \frac{d}{dy}(f_x(x, y))$$

Define a function for $\frac{\partial^2 f}{\partial y \partial x} = f_{yx}$ and display the definition:

$$f_{yx}(x, y) := \frac{d}{dx}(f_x(x, y))$$
Note that for $f(x, y)$ above, the mixed partial derivatives, $f_{xy}$ and $f_{yx}$ are equal:

$$f_{xy}(x, y) = f_{yx}(x, y)$$

This is the case when the mixed partial derivatives of $f(x_0, y_0)$ exist and are continuous in a (possibly small) open disk around the point (Clairaut’s Theorem).

Partial derivatives for functions of more than two variables are calculated in the same manner.

The graph of $f(x, y) = \sqrt{x^2 + y^2}$: