# Using the Fourier Series Library (fourierlib.tns)

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Typeset in LaTeX.



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Most of the examples are from the non-copyrighted book *Fourier Series* by Georgi P. Tolstov, translated from the Russian by Richard A. Silverman, and published by Dover Publications, Inc.

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# 1 Purpose

The functions in `fourierlib` generate fourier series, fourier sine series, and fourier cosine series. The library can be installed and used with TI-Nspire CX CAS, TI-Nspire CAS Student Software, TI-Nspire CAS Teacher Software, and TI-Nspire CAS App for the iPad.

`fourierlib` is free software: you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the Free Software Foundation, either version 3 of the License, or any later version. Visit `https://www.gnu.org/licenses/gpl.htm` to view the License.

# 2 Restrictions/Limitations

Complex fourier series are not supported.

# 3 Library History

Library Version: 1.00
Date Created: 30August2018
Last Modified: 18October2018

# 4 Functions

The functions in the library and examples of using them are described in the following paragraphs. The examples shown below are screen-captures of commands and their results from TI-Nspire CAS Student Software.

## 4.1 help()

Displays usage information for the functions in the library.

### 4.1.1 Example

Select the help function in the library pane to add it to a calculator page, then execute the function:

```
fourierlib\help()
    Help for the fourier library.
    ============================================
    Description:
    Generate fourier series for approximating functions
    Author: Forest W. Arnold
    Version: 1.0
    Date: 15August2018
    Copyright 2018 Forest W. Arnold
```

Scroll to display information about a library function:

```
    1. fourier(function_name,maxival,numterms)
    Purpose:
      calculate a fourier series consisting of 'numterms' non−zero terms
      to approximate the function named 'function_name' over the interval
      [−maxival,maxival].
    Input Arguments:
      function_name − the (string) name of the function to approximate.
      maxival − the largest value of the interval [−maxival,maxival]
      numterms − the number of non−zero terms of the series to generate.
    Returns:
      A fourier series approximating the named function or displays an
       error message and returns a null if the domain of the function does
       not match the interval of integration.
    ----
    2. fouriersin(function_name,maxival,numterms)
```

## 4.2 fourier(function_name,maxival,numterms)

Calculate a fourier series consisting of 'numterms' non-zero terms to approximate the function named 'function_name' over the interval [minival,maxival] where minival equals -maxival.

Input arguments:

> function_name - the (string) name of the function to approximate with a fourier series.
>
> maxival - the maximum value of the interval from -maxival to maxival for the approximation. These are the lower and upper limits of the integral for the ap-

proximation. The lower and upper values of the domain of the function to approximate must include or span these limits.

numterms - the desired number of non-zero terms in the approximating series.

Returns:

A Fourier series approximation for the function named 'function_name'.

**Note:** If the interval [-maxival,maxival] is larger than the domain of the function (for example, the domain of a piecewise-defined function), this function displays an error message and returns an empty string.

#### 4.2.1 Example 1

Expand the even function $f(x) = x^2, -pi \leq x \leq pi$ in Fourier series.

The definition of the function and the command to expand the function in a calculator application is

$ex1(x):=x^2 | {}^{-}\pi \leq x \leq \pi$ *Done*

ser1:=fourierlib\fourier("ex1",π,8)

$$\frac{{}^{-}4\cdot \cos(7\cdot x)}{49}+\frac{\cos(6\cdot x)}{9}-\frac{4\cdot \cos(5\cdot x)}{25}+\frac{\cos(4\cdot x)}{4}-\frac{4\cdot \cos(3\cdot x)}{9}+\cos(2\cdot x)-4\cdot \cos(x)+\frac{\pi^2}{3}$$

The graph of the function and its Fourier series:

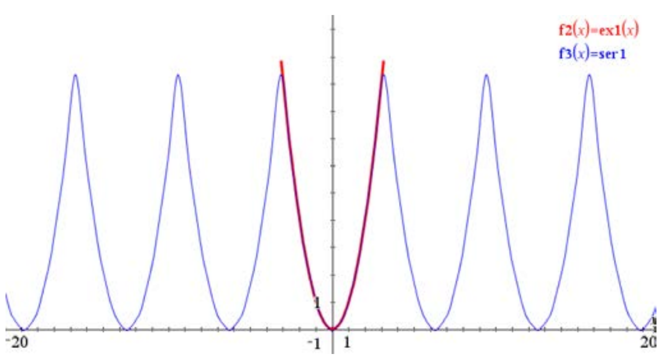


Figure 1: Periodic Extension of $f(x) = x^2$

### 4.2.2 Example 2

Expand the even function $f(x) = |x|, -pi \leq x \leq pi$ in Fourier series:

$ex2(x) := |x| \mid {}^{-}\pi \leq x \leq \pi$ *Done*

$ser2 := fourierlib\backslash fourier(\text{"ex2"}, \pi, 6)$

$◂02598448 \cdot \cos(7 \cdot x) - 0.05092958 \cdot \cos(5 \cdot x) - 0.14147106 \cdot \cos(3 \cdot x) - 1.2732395 \cdot \cos(x) + \frac{\pi}{2}$

The graph of the function and its Fourier series:

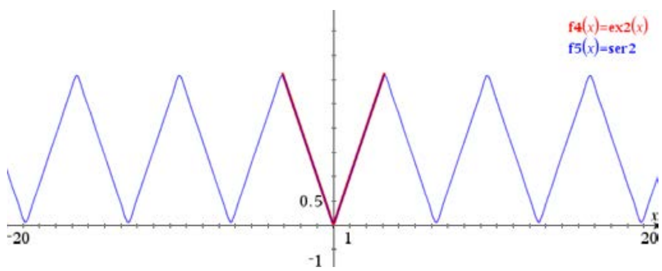


Figure 2: Periodic Extension of $f(x) = |x|$

### 4.2.3 Example 3

Expand the odd function $f(x) = x, -pi < x < pi$ in Fourier series.

$ex3(x) := x \mid {}^{-}\pi < x < \pi$ *Done*

$ser3 := fourierlib\backslash fourier(\text{"ex3"}, \pi, 6)$

$$\frac{{}^{-}\sin(6 \cdot x)}{3} + \frac{2 \cdot \sin(5 \cdot x)}{5} - \frac{\sin(4 \cdot x)}{2} + \frac{2 \cdot \sin(3 \cdot x)}{3} - \sin(2 \cdot x) + 2 \cdot \sin(x)$$

The graph of the function and its Fourier series:

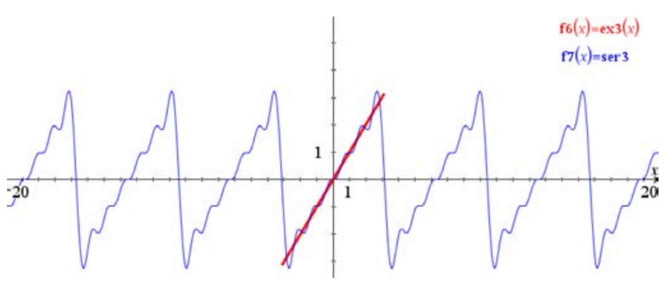


Figure 3: Periodic Extension of $f(x) = x$

### 4.2.4 Example 4

Expand the function $f(x) = e^{-x}, -2 \leq x \leq 2$ in Fourier series:

$ex4(x) := \mathbf{e}^{-x} | -2 \leq x \leq 2$ *Done*

$ser4 := fourierlib\backslash fourier(\text{"ex4"}, 2, 15)$

$$\dots\frac{\sinh(2)\cdot\cos\left(\frac{\pi\cdot x}{2}\right)}{\pi^2+4} - \frac{2\cdot\sinh(2)\cdot\pi\cdot\sin\left(\frac{\pi\cdot x}{2}\right)}{\pi^2+4} + \frac{\sinh(2)\cdot\cos(\pi\cdot x)}{\pi^2+1} + \frac{\sinh(2)\cdot\pi\cdot\sin(\pi\cdot x)}{\pi^2+1} + \frac{\sinh(2)}{2}$$

The graph of the function and its Fourier series:

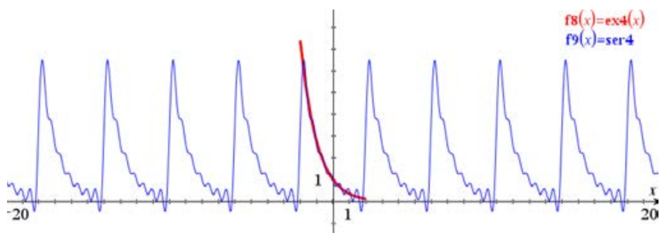


Figure 4: Periodic Extension of $f(x) = e^{-x}$

### 4.2.5 Example 5

Expand the odd function $f(x) = \begin{cases} -1, & -1 \leq x < 0 \\ 1, & 0 < x \leq 1 \end{cases}$ in Fourier series.

$ex5(x) := \begin{cases} -1, & -1 \leq x < 0 \\ 1, & 0 < x \leq 1 \end{cases}$ *Done*

$ser5 := fourierlib\backslash fourier(\text{"ex5"}, 1, 9)$

$\dots 9136\cdot\sin(7\cdot\pi\cdot x) + 0.25464791\cdot\sin(5\cdot\pi\cdot x) + 0.42441318\cdot\sin(3\cdot\pi\cdot x) + 1.2732395\cdot\sin(\pi\cdot x)$

The graph of the function and its Fourier series:

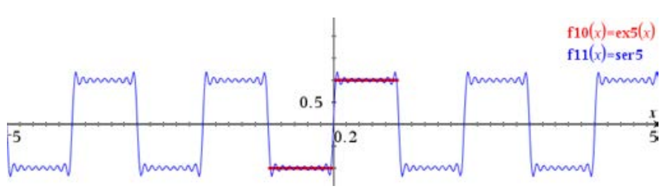


Figure 5: Periodic Extension of $f(x)$

## 4.3 fouriercos(function_name,maxival,numterms)

Calculate a fourier cosine series consisting of 'numterms' non-zero terms to approximate the function named 'function_name' over the interval [0,maxival].

Input arguments:

> function_name - the (string) name of the function to approximate with a fourier cosine series.
>
> maxival - the maximum value of the interval from 0 to maxival for the approximation. These are the lower and upper limits of the integral for the approximation. The lower and upper values of the domain of the function to approximate must include or span these limits.
>
> numterms - the desired number of non-zero terms in the approximating series.

Returns:

> A Fourier cosine series with even extension approximation for the function named 'function_name'.

**Note:** If the interval [0,maxival] is larger than the domain of the function (for example, the domain of a piecewise-defined function), this function displays an error message and returns an empty string.

### 4.3.1 Example 1

Expand $f(x) = \begin{cases} \cos(\frac{\pi x}{2}), & 0 \leq x \leq 1 \\ 0, & 1 < x < 2 \end{cases}$ in Fourier cosine series.

$$ex1(x):=\begin{cases} \cos\left(\frac{\pi \cdot x}{2}\right), & 0\leq x\leq 1 \\ 0, & 1<x<2 \end{cases} \qquad \textit{Done}$$

$$ser1:=fourierlib\backslash fouriercos(\text{"ex1"},2,6)$$

$$\ldots 01818914\cdot \cos(3\cdot \pi\cdot x)-0.04244132\cdot \cos(2\cdot \pi\cdot x)+0.5\cdot \cos\left(\frac{\pi\cdot x}{2}\right)+0.21220659\cdot \cos(\pi\cdot x)+\frac{1}{\pi}$$

The graph of the function and its Fourier cosine series:

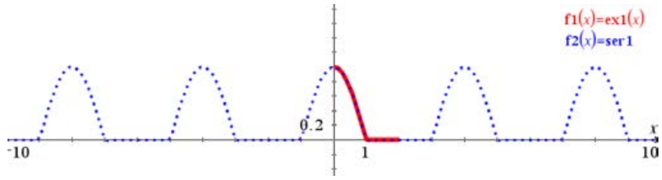


Figure 6: Even Periodic Extension of $f(x)$

### 4.3.2 Example 2

Expand $f(x) = \begin{cases} x, & 0 \leq x \leq \frac{3}{2} \\ 3-x, & \frac{3}{2} < x \leq 3 \end{cases}$ in Fourier cosine series.

$$ex2(x) := \begin{cases} x, & 0 \leq x \leq \frac{3}{2} \\ 3-x, & \frac{3}{2} < x \leq 3 \end{cases}$$ *Done*

*ser2*:=*fourierlib**fouriercos*("ex2",3,6)

$$\blacktriangleleft \frac{4\cdot\pi\cdot x}{3}\Big) - 0.02431708\cdot\cos\left(\frac{10\cdot\pi\cdot x}{3}\right) - 0.06754746\cdot\cos(2\cdot\pi\cdot x) - 0.6079271\cdot\cos\left(\frac{2\cdot\pi\cdot x}{3}\right) + \frac{3}{4}$$

The graph of the function and its Fourier cosine series:

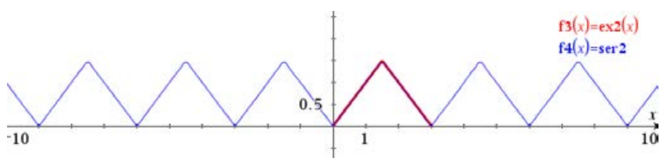


Figure 7: Even Periodic Extension of $f(x)$

## 4.4 fouriersin(function_name,maxival,numterms)

Calculate a fourier sine series consisting of 'numterms' non-zero terms to approximate the function named 'function_name' over the interval [0,maxival].

Input arguments:

> function_name - the (string) name of the function to approximate with a fourier sine series.
>
> maxival - the maximum value of the interval from 0 to maxival for the approximation. These are the lower and upper limits of the integral for the approxima-

tion. The lower and upper values of the domain of the function to approximate must include or span these limits.

numterms - the desired number of non-zero terms in the approximating series.

Returns:

A Fourier sine series with odd extension approximation for the function named 'function_name'.

**Note:** If the interval [0,maxival] is larger than the domain of the function (for example, the domain of a piecewise-defined function), this function displays an error message and returns an empty string.

### 4.4.1 Example 1

Expand $f(x) = \begin{cases} x, & 0 \le x \le \frac{3}{2} \\ 3-x, & \frac{3}{2} < x \le 3 \end{cases}$ in Fourier sine series.

$$ex1(x):=\begin{cases} x, & 0\le x\le \frac{3}{2} \\ 3-x, & \frac{3}{2}<x\le 3 \end{cases}$$ Done

```
ser1:=fourierlib\fouriersin("ex1",3,6)
```

$$\cdots 1335\cdot \sin\left(\frac{7\cdot \pi\cdot x}{3}\right)+0.04863417\cdot \sin\left(\frac{5\cdot \pi\cdot x}{3}\right)+1.2158542\cdot \sin\left(\frac{\pi\cdot x}{3}\right)-0.13509491\cdot \sin(\pi\cdot x)$$

The graph of the function and its Fourier sine series:

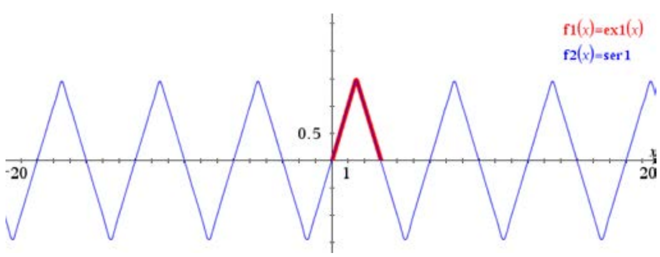


Figure 8: Odd Periodic Extension of $f(x)$

### 4.4.2 Example 2

Expand $f(x) = \begin{cases} \cos(\frac{\pi x}{2}), & 0 \leq x \leq 1 \\ 0, & 1 < x < 2 \end{cases}$ in Fourier sine series.

$$ex2(x) := \begin{cases} \cos\left(\frac{\pi \cdot x}{2}\right), & 0 \leq x \leq 1 \\ 0, & 1 < x < 2 \end{cases} \qquad \textit{Done}$$

ser2:=fourierlib\fouriersin("ex2",2,10)

$$\text{◂}6527 \cdot \sin(2 \cdot \pi \cdot x) + 0.31830989 \cdot \sin\left(\frac{3 \cdot \pi \cdot x}{2}\right) + 0.31830989 \cdot \sin\left(\frac{\pi \cdot x}{2}\right) + 0.42441318 \cdot \sin(\pi \cdot x)$$

The graph of the function and its Fourier sine series:

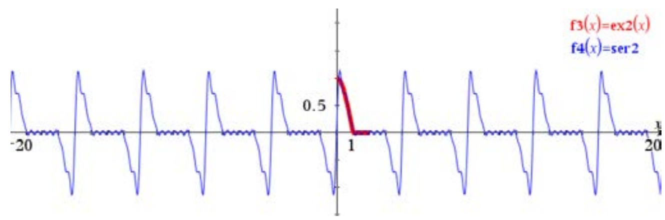


Figure 9: Odd Periodic Extension of $f(x)$

## 4.5 fouriercoscoef(function_name,maxival,index)

Calculate a specified coefficient of a cosine term of a fourier series for the function named 'function_name' over the interval [-maxival,maxival].

Input arguments:

> function_name - the (string) name of the function.
>
> maxival - the maximum value of the interval from -maxival to maxival for the approximation. These are the lower and upper limits of the integral for the approximation. The lower and upper values of the domain of the function to approximate must include or span these limits.
>
> index - the index of the desired coefficient. Indices for cosine terms begin with 0.

Returns:

> The coefficient of a cosine term of the Fourier series for the function named 'function_name'. The returned **first** term is divided by 2: the first cosine coefficient = $\frac{a_0}{2}$.

**Note:** If the interval [-maxival,maxival] is larger than the domain of the function (for example, the domain of a piecewise-defined function), this function displays an error message and returns an empty string.

### 4.5.1 Example 1

Find the first five cosine coefficients for the Fourier series approximation for the even function $x^2, -\pi \le x \le \pi$:

| | |
|---|---|
| $ex1(x):=x^2 \mid -\pi \le x \le \pi$ | *Done* |
| For $i$,0,4:$a[i+1]$:=*fourierlib\fouriercoscoef*("ex1",π,$i$):EndFor | $\frac{1}{4}$ |
| $a$ | $\left\{\frac{\pi^2}{3},{}^-4,1,\frac{{}^-4}{9},\frac{1}{4}\right\}$ |

### 4.5.2 Example 2

Find the first five cosine coefficients for the Fourier series approximation for the odd function $x, -\pi \le x \le \pi$:

| | |
|---|---|
| $ex2(x):=x \mid {}^-\pi < x < \pi$ | *Done* |
| For $i$,0,4:$a[i+1]$:=*fourierlib\fouriercoscoef*("ex2",π,$i$):EndFor | 0 |
| $a$ | {0,0,0,0,0} |

**Note:** all the cosine coefficients for an **odd** function equal 0.

## 4.6 fouriersincoef(function_name,maxival,index)

Calculate a specified coefficient of a sine term of a fourier series for the function named 'function_name' over the interval [-maxival,maxival].

Input arguments:

> function_name - the (string) name of the function.
>
> maxival - the maximum value of the interval from -maxival to maxival for the approximation. These are the lower and upper limits of the integral for the approximation. The lower and upper values of the domain of the function to approximate must include or span these limits.
>
> index - the index of the desired coefficient. Indices for sine terms begin with 1.

Returns:

> The coefficient of a sine term of the Fourier series for the function named 'function_name'.

**Note:** If the interval [-maxival,maxival] is larger than the domain of the function (for example, the domain of a piecewise-defined function), this function displays an error message and returns an empty string.

### 4.6.1 Example 1

Find the first five sine coefficients for the Fourier series approximation for the even function $x^2, -\pi \leq x \leq \pi$:

| | |
|---|---|
| $ex1(x):=x^2 \mid {}^-\pi \leq x \leq \pi$ | *Done* |
| $b[1]$:=0:For $i$,1,4:$b[i+1]$:=*fourierlib\fouriersincoef*("ex1",π,$i$):EndFor | 0 |
| $b$ | {0,0,0,0,0} |

**Note:** all the sine coefficients for an **even** function equal 0.

### 4.6.2 Example 2

Find the first five sine coefficients for the Fourier series approximation for the odd function $x, -\pi \leq x \leq \pi$:

| | |
|---|---|
| $ex2(x):=x \mid {}^-\pi < x < \pi$ | *Done* |
| $b[1]$:=0:For $i$,1,4:$b[i+1]$:=*fourierlib\fouriersincoef*("ex2",π,$i$):EndFor | $\frac{-1}{2}$ |
| $b$ | $\left\{0,2,-1,\frac{2}{3},\frac{-1}{2}\right\}$ |

### 4.6.3 Example 3

Use the first four Fourier cosine and sine coefficients to construct a Fourier series for the function $f(x) = \begin{cases} -1, & -1 \leq x < 0 \\ x, & 0 < x \leq 1 \end{cases}$.

Define the piecewise function:

| | |
|---|---|
| $ex3(x):=\begin{cases} -1, & -1 \leq x < 0 \\ x, & 0 < x \leq 1 \end{cases}$ | *Done* |

Calculate the cosine coefficients:

| | |
|---|---|
| For $i$,0,3:$a[i+1]$:=*fourierlib\fouriercoscoef*("ex3",1,$i$):EndFor | ⁻0.02251582 |
| $a$ | $\left\{\frac{-1}{4}, {}^-0.20264237, 0, {}^-0.02251582, 0\right\}$ |

Calculate the sine coefficients:

$b[1]$:=0:For $i$,1,3:$b[i+1]$:=*fourierlib\fouriersincoef*("ex3",1,$i$):EndFor 0.31830989

$b$ {0,0.95492966,⁻0.15915494,0.31830989,⁻0.07957747}

Use the formula

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{\pi n x}{l}\right) + b_n \sin\left(\frac{\pi n x}{l}\right)\right)$$

where $\frac{a_0}{2} = a[1]$ and $l$ = upper interval value to construct the series (note that the $a_0$ term returned by `fouriercoscoef()` has already been divided by 2):

$l$:=1 1

$$fpoly:=a[1]+\sum_{i=2}^{3}\left(a[i]\cdot\cos\left(\frac{(i-1)\cdot\pi\cdot x}{l}\right)+b[i]\cdot\sin\left(\frac{(i-1)\cdot\pi\cdot x}{l}\right)\right)$$

$$^{-}0.15915494\cdot\sin(2\cdot\pi\cdot x)-0.20264237\cdot\cos(\pi\cdot x)+0.95492966\cdot\sin(\pi\cdot x)-\frac{1}{4}$$

The series returned by the `fourier()` function is:

*fourierlib\fourier*("ex3",1,4)

$$^{-}0.15915494\cdot\sin(2\cdot\pi\cdot x)-0.20264237\cdot\cos(\pi\cdot x)+0.95492966\cdot\sin(\pi\cdot x)-\frac{1}{4}$$

## 4.7 isevenfunc(function_name)

Determine if the function named 'function_name' is an **even** function.

Input arguments:

> function_name - the (string) name of the function.

Returns:

> `true` if the named function is an even function and `false` if the function is not an even function.

An even function is a function such that $f(x) = f(-x)$. The graph of an even function is symmetric with respect to the $y-axis$.

### 4.7.1 Example 1

$ex1(x)$:=$x^2$ *Done*

*fourierlib\isevenfunc*("ex1") true

#### 4.7.2 Example 2

*ex2*(*x*):=*x* *Done*

*fourierlib\isevenfunc*("ex2") false

#### 4.7.3 Example 3

*ex3*(*x*):=$\mathbf{e}^{x} \cdot \cos(x)$ *Done*

*fourierlib\isevenfunc*("ex3") false

Note that $ex3(x) = e^x \cdot \cos(x)$ is neither even nor odd.

### 4.8 isoddfunc(function_name)

Determine if the function named ’function_name’ is an **odd** function.

Input arguments:

> function_name - the (string) name of the function.

Returns:

> `true` if the named function is an odd function and `false` if the function is not an odd function.

An odd function is a function such that $f(-x) = -f(x)$. The graph of an odd function is symmetric with respect to the *origin*.

#### 4.8.1 Example 1

*ex1*(*x*):=$x^2$ *Done*

*fourierlib\isoddfunc*("ex1") false

#### 4.8.2 Example 2

*ex2*(*x*):=*x* *Done*

*fourierlib\isoddfunc*("ex2") true

#### 4.8.3 Example 3

*ex3*(*x*):=$\mathbf{e}^{x} \cdot \cos(x)$ *Done*

*fourierlib\isoddfunc*("ex3") false

Note that $ex3(x) = e^x \cdot \cos(x)$ is neither even nor odd.